Examples of Binomial Distribution:

(1) A merchant's file of 20 accounts contains 6 delinquent and 14 non-delinquent accounts. An auditor randomly selects 5 of these accounts for examination.

(i) what is the probability that the auditor finds exactly 2 delinquent accounts?

(ii) Find the expected number of delinquent accounts in the sample selected.

Solution

p = probability of a delequent account $= \frac{6}{20} = 0.3$.

$\Rightarrow q = 1 - p = 1 - 0.3 = 0.7$.

$P(r)$: probability the auditor finds (r) a deliquent account in 5 random accounts.

$$P(r) = {}^nC_r p^n q^{n-r}$$
$$= {}^5C_r (0.3)^r (0.7)^{5-r}$$

(i) The probability that the auditor finds exactly 2 delinquent account.

$$P(2) = {}^5C_2 (0.3)^2 (0.7)^3$$
$$= \frac{5 \times 4}{2} \times 0.09 \times 0.343$$
$$= 0.3087$$

(ii) The expected number of delinquent accounts in the selected sample of $n = 5$ is $n \times p = 5 \times 0.3 = 1.5$

(2) With the usual notations, find p for a binomial random variable X if $n = 6$ and if

$$9P(X=4) = P(X=2).$$

<u>Solution</u>

$$P(r) = P(X=r) = {}^6C_r\, p^r q^{6-r}$$

Given $\quad 9P(X=4) = P(X=2)$

$$9\,{}^6C_4\, p^4 q^{6-4} = {}^6C_2\, p^2 q^{6-2}$$

$\Rightarrow 9\,{}^6C_4\, p^4 q^2 = {}^6C_2\, p^2 q^4 \quad [\because {}^6C_4 = {}^6C_{6-4} = {}^6C_2]$

$\Rightarrow \quad 9p^2 = q^2$

$\Rightarrow \quad 9p^2 = (1-p)^2 = 1 + p^2 - 2p$

$\Rightarrow \quad 8p^2 + 2p - 1 = 0$

$\Rightarrow \quad 8p^2 + 4p - 2p - 1 = 0$

$\Rightarrow 4p(2p+1) - 1(2p+1) = 0$

$\Rightarrow (4p-1)(2p+1) = 0$

$\Rightarrow 4p - 1 = 0 \qquad \Rightarrow 2p + 1 = 0$

$\Rightarrow p = 1/4 \qquad \Rightarrow p = -1/2$ (probability cannot be negative).

(Ans)

(3) Assume that half the population is vegetarian so that the chance of an individual being a vegetarian is $\frac{1}{2}$. Assuming that 100 investigators each take sample of 10 individuals to see whether they are vegetarians, how many investigators would you expect to report that three people or less were vegetarian?

Solution

Given $n = 10$,

p = probability that an individual is a vegeterian $= \frac{1}{2}$

$q = 1 - p = \frac{1}{2}$.

$$P(r) = {}^{10}C_r p^r q^{10-r}$$
$$= {}^{10}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r}$$
$$= {}^{10}C_r \left(\frac{1}{2}\right)^{10}$$
$$= \frac{1}{1024} {}^{10}C_r$$

The probability that in a sample of 10 the number of vegetarians will 3 or less

$$P(0) + P(1) + P(2) + P(3) = \frac{1}{1024}\left[{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3\right]$$
$$= \frac{1}{1024}\left[1 + 10 + 45 + 120\right]$$
$$= \frac{176}{1024}$$
$$= \frac{11}{64}.$$

Out of 100 investigators

$$100 \times \frac{11}{64} = \frac{275}{16}$$
$$= 17.2 \simeq 17. \text{ (ans)}$$

Reference: Fundamentals of Statistics by S.C. Gupta